Noise equivalent power (and energy resolution) of transition-edge sensors with complex thermal models

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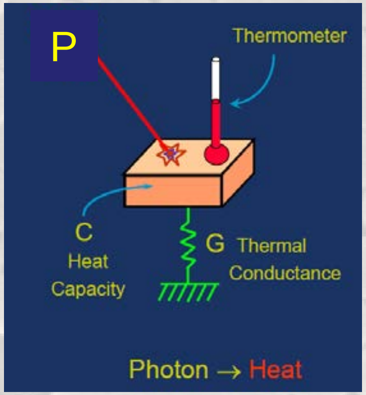
Thermal Properties of Nanostructures

Intro-I

- bolometer performance is most often analyzed in terms of the simplest thermal model with a single heat capacity connected to the bath



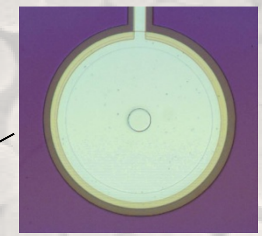
- Equations are fairly simple and well known, see e.g. the authoritative reviews by McCammon and Irwin and Hilton in Cryogenic Particle Detection, Ed. Ch. Enss, Springer 2005

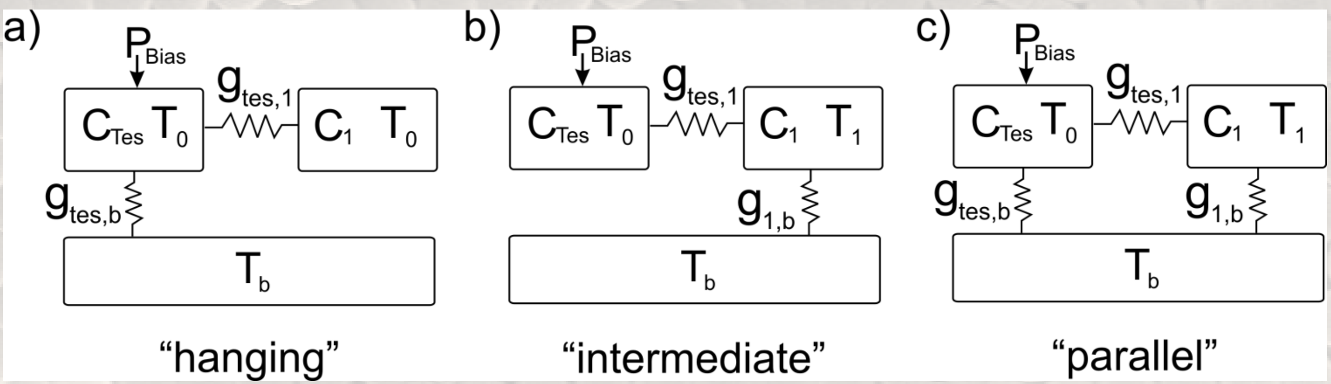
- For TES bolometers in the high loop gain limit, a particularly simple equation for the NEP, limited by *thermodynamic energy fluctuations* (phonon noise) between the TES and the bath:

$$NEP(\omega) = \sqrt{4k_B T_0^2 G \times F(T_0, T_{bath})}$$

In reality, though, many types of TESes have been experimentally shown to have a more complex thermal circuit

H. F. C. Hoevers *et al.*, Appl. Phys. Lett. 77, 4422 (2000).
B. L. Zink *et al.,* Appl. Phys. Lett. 89, 124101 (2006).
T. Saab *et al.,* , J. Appl. Phys. 102, 104502 (2007).
Y. Zhao *et al.,* , IEEE Trans. Appl.Supercond. 21, 227 (2011).
M. R. J. Palosaari *et al.,* J. Low Temp. Phys. 167, 129 (2012).
K. M. Kinnunen, M. R. J. Palosaari, and I. J. Maasilta, J. Appl. Phys. 112, 034515 (2012).





A lot of previous theoretical work done on understanding the noise and responsivity, complex impedance and NEP of ”thermally challenged” TESes

H. F. C. Hoevers *et al.*, Appl. Phys. Lett. 77, 4422 (2000).
B. L. Zink *et al.,* Appl. Phys. Lett. 89, 124101 (2006).
M. Galeazzi and D. McCammon, J. Appl. Phys. 93, 4856 (2003).
E. Figueroa-Feliciano, J. Appl. Phys. 99, 114513 (2006).
J. W. Appel and M. Galeazzi, Nucl. Instrum. Meth. A 562, 272 (2006).
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D. J. Goldie *et al.*, J. Appl. Phys. 105, 074512 (2009).
Y. Zhao, Ph.D. thesis, Princeton University, 2010.
D. A. Bennett *et al.*, Appl. Phys. Lett. 97, 102504 (2010).
M. A. Lindeman *et al.,* IEEE Trans. Appl. Supercond. 21, 254 (2011).
I. J. Maasilta, AIP Advances 2, 042110 (2012).

Intro-II

- Common assumption or belief: The phonon noise limited NEP of a single block bolometer is the best we could ever do for a given G to bath.

$$NEP(\omega) = \sqrt{4k_B T_0^2 G \times F(T_0, T_{bath})}$$

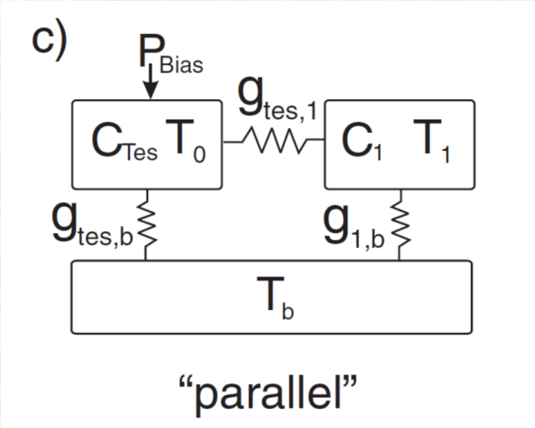
- It is based on the known fact that complex thermal models introduce ”excess” or ”additional” thermodynamic noise.

- Question: Is this true? If so, how do we approach that limit ?

Thermal Properties of Nanostructures

Intro-II

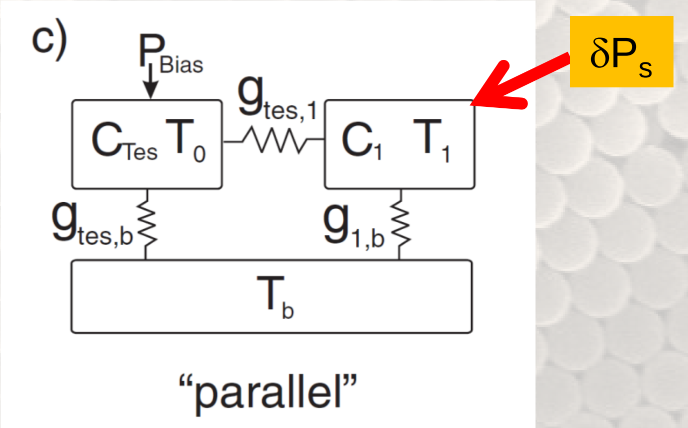
- Focus here on a general two-block model, using analytical formulations for noise and Z derived before



I. J. M, AIP Advances 2, 042110 (2012)

- Important point: consider the case where C1 is the absorber, i.e. small signal power does not couple directly into the TES

- Focus here on a general two-block model, using analytical formulations for noise and Z derived before

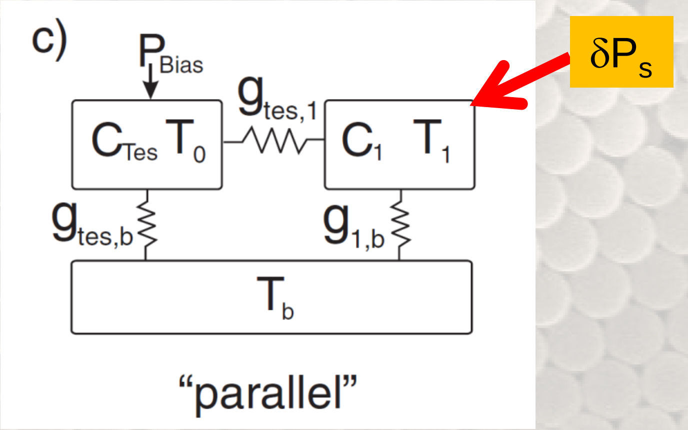


I. J. M, AIP Advances 2, 042110 (2012)

•Important point 1: consider the case where C1 is the absorber, i.e. small signal power does not couple directly into the TES

=> Two blocks are not equivalent!

- Focus here on a general two-block model, using analytical formulations for noise and Z derived before



I. J. M, AIP Advances 2, 042110 (2012)

•Important point 2: Joule power P_{bias} is dissipated through the whole network, *T1 is thus determined self-consistently* by the g:s

Thermal Properties of Nanostructures

Intro-II

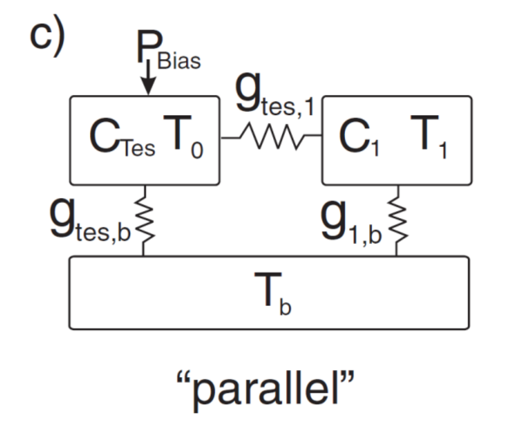
- Can derive the following NEP contributions (three thermal one Johnson), all thermal links *non-diffusive*:

$$NEP_{1,b}(\omega) = \sqrt{2k_B[g_{1,b}(T_1)T_1^2 + g_{1,b}(T_b)T_b^2]}$$

$$NEP_{tes,1}(\omega) = \sqrt{2k_B[g_{tes,1}(T_0)T_0^2 + g_{tes,1}(T_1)T_1^2]}\frac{1}{A}\sqrt{\frac{g_{1,b}(T_1)^2}{[g_{1,b}(T_1) + g_{tes,1}(T_1)]^2} + \omega^2\tau_1^2}$$

$$NEP_{tes,b}(\omega) = \sqrt{2k_B[g_{tes,b}(T_0)T_0^2 + g_{tes,b}(T_b)T_b^2]}\frac{1}{A}\sqrt{1 + \omega^2\tau_1^2}$$

$$NEP_J(\omega) = \frac{V_\omega}{I_0}\left|\frac{Z_{tes} + R_0}{Z_{tes} - R_0(1+\beta)}\right|\frac{1}{A}\sqrt{1 + \omega^2\tau_1^2}$$



$$A = \frac{g_{tes,1}(T_1)}{g_{1,b}(T_1) + g_{tes,1}(T_1)}$$

$$\tau_1 = C_1/(g_{tes,1}(T_1) + g_{1,b}(T_1))$$

$$V_\omega = \sqrt{4k_BT_0R_0(1+2\beta)}$$

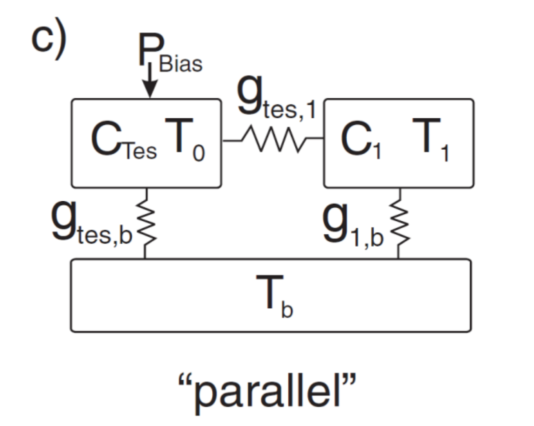
Assuming $P \sim T^4$ for both $g_{tes,1}$ and $g_{1,b}$, get also

$$T_1^4 = AT_0^4 + (1-A)T_b^4$$

Thermal Properties of Nanostructures

Intro-II

- Focus here on bolometers the high loop gain *L* limit
- As $NEP_J \sim 1/L$, we can disregard it
- Also, look at low-f response, i.e. below time constant τ_1
- <u>Study two cases</u>



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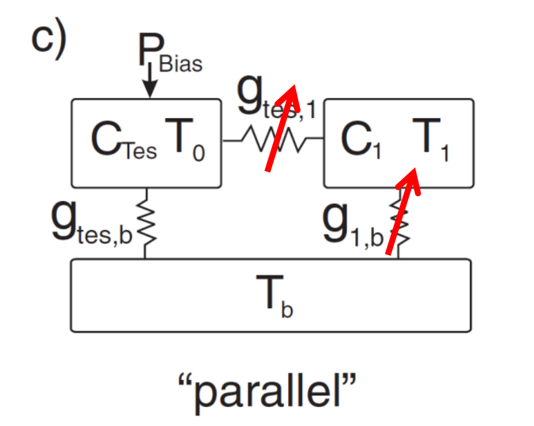

- <u>Study two cases</u>
  1) Keep coupling between TES and bath ($g_{tes,b}$) constant and vary coupling from absorber to TES and bath ($g_{tes,1}$ and $g_{1,b}$)

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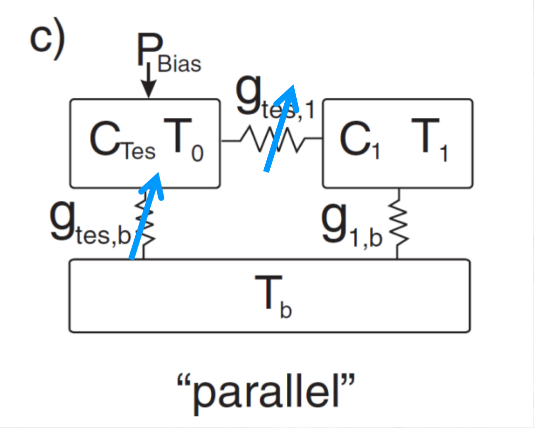

- Also, look at low-f response,
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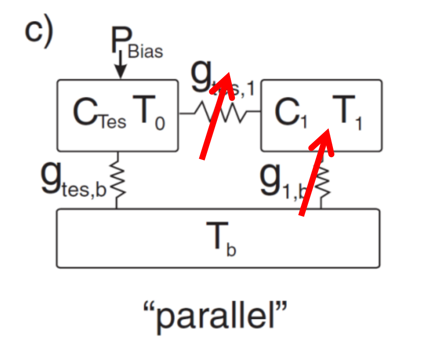
- <u>Study two cases</u>
  1) Keep coupling between TES and bath ($g_{tes,b}$) constant and vary coupling from absorber to TES and bath ($g_{tes,1}$ and $g_{1,b}$)
  2) Keep coupling between absorber and bath ($g_{1,b}$) constant and vary coupling from TES to absorber and bath ($g_{tes,1}$ and $g_{tes,b}$)

Note! In the second case only strength a is constant ($g_{1,b}$ = a T_1^3), and g value actually still scales with T_1

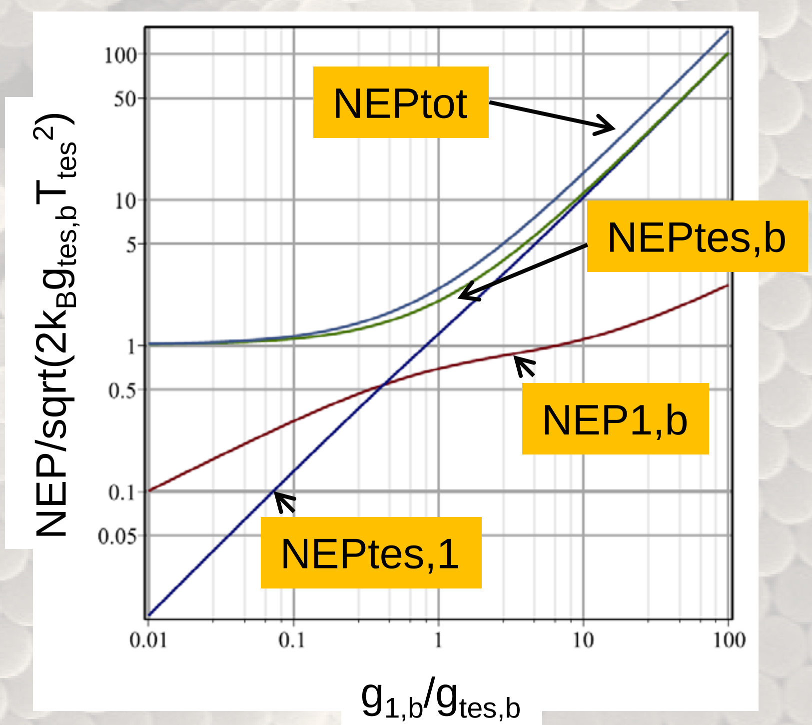
Thermal Properties of Nanostructures



1) vary coupling from absorber to TES and bath

($g_{tes,1}$ and $g_{1,b}$) Ttes = 100 mK, Tbath = 50 mK

$g_{tes,1}/g_{tes,b}$ =1



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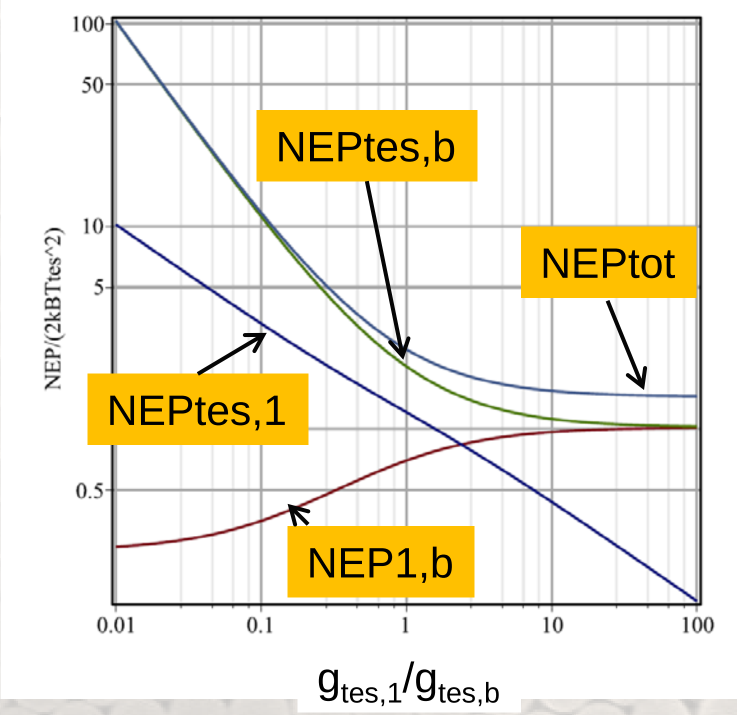

Want low coupling from absorber to bath and high coupling from TES to absorber
=> Approaching simple model !

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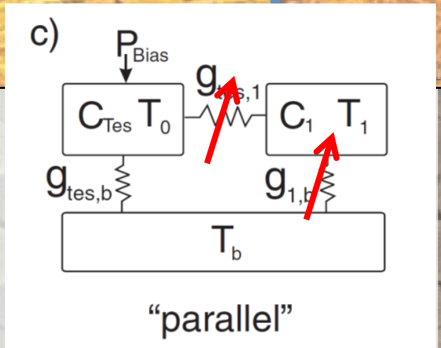
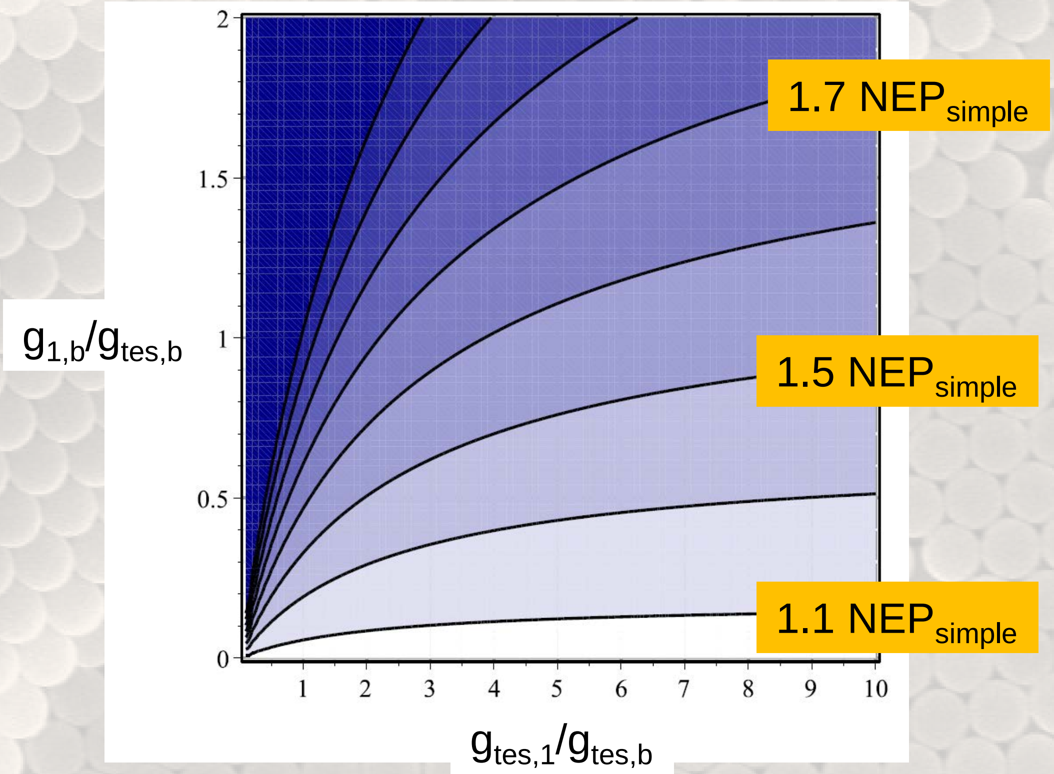


1) vary coupling from absorber to TES and bath

($g_{tes,1}$ and $g_{1,b}$) Ttes = 100 mK, Tbath = 50 mK

Contour plot

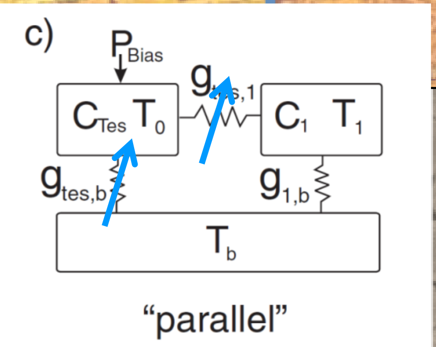


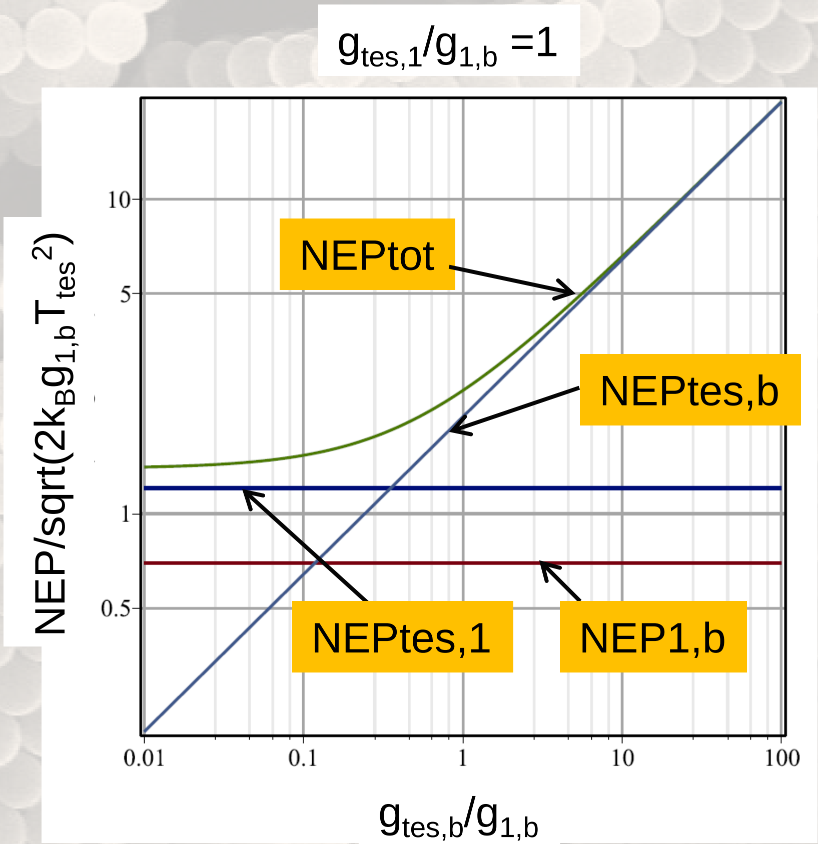
=> g1,b always bad, but stronger gtes,1 can improve situation

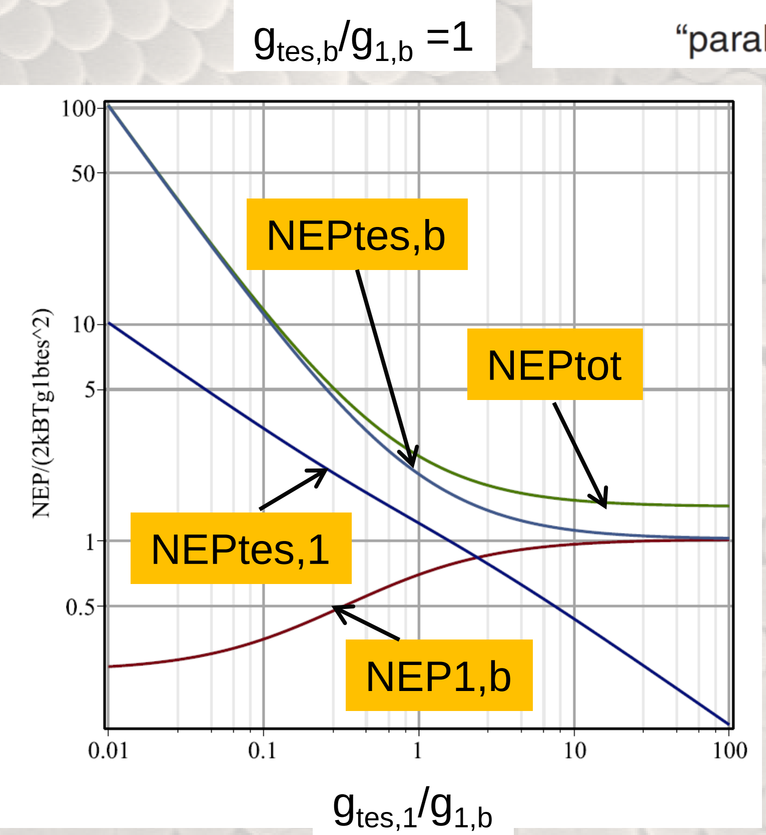
Thermal Properties of Nanostructures

2) vary coupling from TES to absorber and bath

($g_{tes,1}$ and $g_{tes,b}$) Ttes = 100 mK, Tbath = 50 mK







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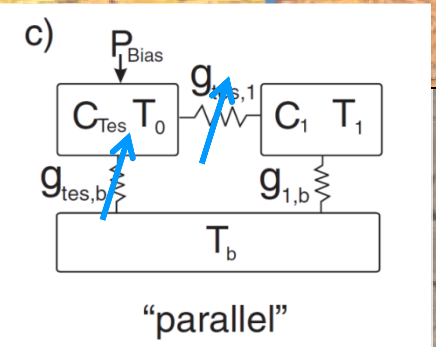
Thermal Properties of Nanostructures



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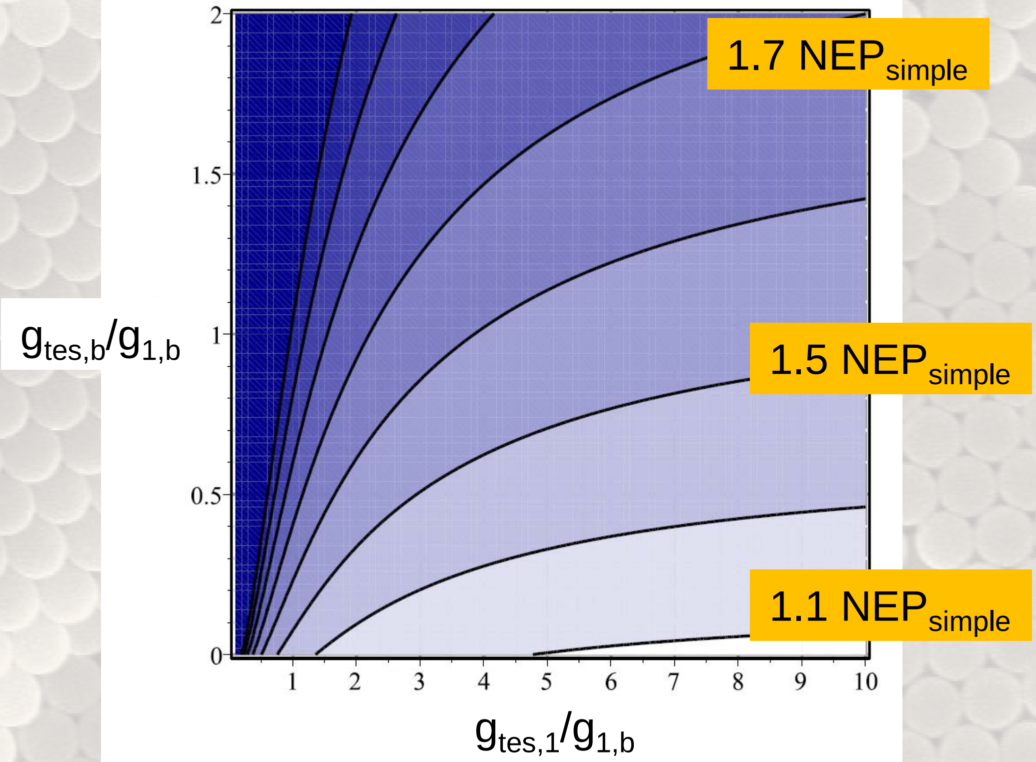

⇒ gtes,b always bad, but stronger gtes,1 can improve situation

⇒ Need higher gtes,1 to improve

Intro-II

- Result: The phonon noise limited NEP of a single block bolometer is the best we could ever do for a given G to bath.

$$NEP(\omega) = \sqrt{4k_B T_0^2 G \times F(T_0, T_{bath})}$$

- We have demonstrated how to mitigate these problems

- Question #2: Are we done? *What if T1 is a free parameter?*

Seems we could beat the simple bolometer if somehow T1 can be kept low !

Conclusions

- We have given compact analytical results for the NEP of TES detectors with a general two heat capacity thermal model, where sensor and absorber are separate elements

- Results were given how to best approach the ideal limit

- Evidence that simple TES limit can perhaps be beaten if include active cooling (passive won't work)