Error Correcting codes for code-division multiplexed TES detectors

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Abstract

Transition edge sensors (TES) have proven to be highly sensitive and versatile X-ray spectrometers. Upcoming missions, including Athena X-IFU, will rely on highly multiplexed TES arrays with good energy resolution. Code-division multiplexing (CDM) can provide the required multiplexing factors and noise performance, but is susceptible to single SQUID failures. Error correcting codes (ECC) can provide redundancy to loss of one or more SQUID readout devices. We have implemented an ECC algorithm for CDM arrays that is scalable and easy implemented in hardware. We will present this algorithm for error correction, as well as the result of laboratory tests to assess algorithm performance at recovering TES channels after a SQUID failure.

Multiplexing Transition Edge Sensors

Multiplexing development is critical to harness TES arrays for next-generation science

- Better noise performance
- Fewer wires between cryogenic stage and room temperature

Time-division multiplexing (TDM)

- Each SQUID reads one TES
- Read one SQUID at a time
- Noise penalty due to SQUID bandwidth

Code-division multiplexing (CDM)

- Each SQUID reads a linear combination of TES
- Read one SQUID at a time
- Every TES is read all the time, **no noise penalty**
- Compatible with existing TDM architectures

Basic scheme for code-division multiplexing

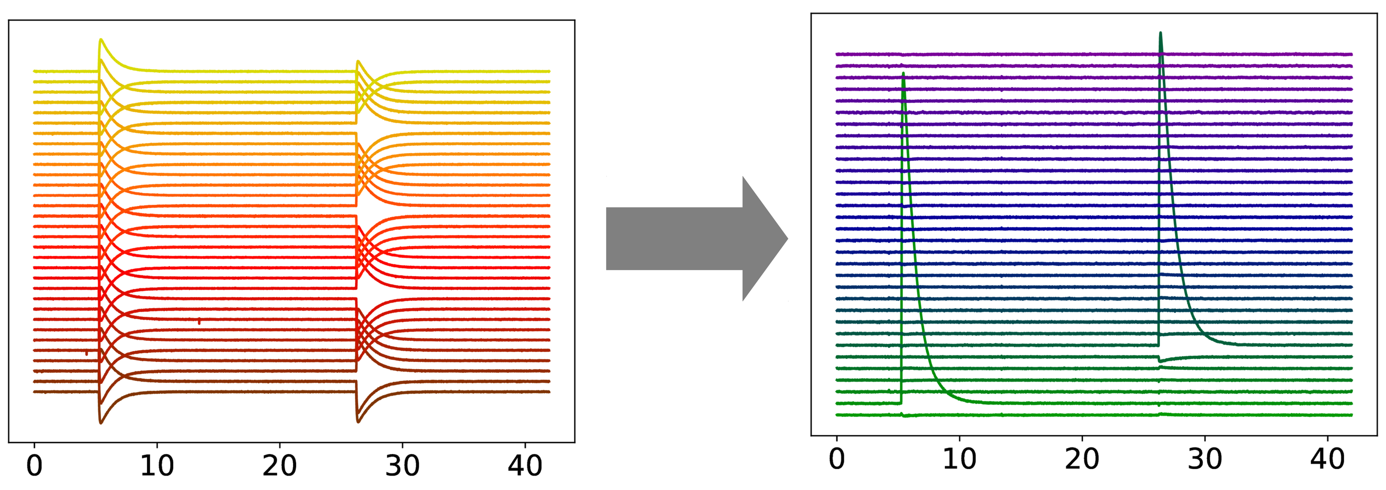
TES signals are multiplexed via a Walsh encoding matrix

$$\begin{pmatrix}\text{Squid1} & \text{Squid2} & \text{Squid3} & \text{Squid4}\end{pmatrix} = \begin{pmatrix}-1 & -1 & -1 & -1\\ -1 & 1 & -1 & 1\\ -1 & -1 & 1 & 1\\ -1 & 1 & 1 & -1\end{pmatrix}\begin{pmatrix}\text{TES1}\\ \text{TES2}\\ \text{TES3}\\ \text{TES4}\end{pmatrix}$$

TES signals are recovered via the inverse (decoding) matrix

$$\begin{pmatrix}\text{TES1} & \text{TES2} & \text{TES3} & \text{TES4}\end{pmatrix} = \begin{pmatrix}-0.25 & -0.25 & -0.25 & -0.25\\ -0.25 & 0.25 & -0.25 & 0.25\\ -0.25 & -0.25 & 0.25 & 0.25\\ -0.25 & 0.25 & 0.25 & -0.25\end{pmatrix}\begin{pmatrix}\text{Squid1}\\ \text{Squid2}\\ \text{Squid3}\\ \text{Squid4}\end{pmatrix}$$

Examples of CDM encoded and decoded TES data



The raw signal from each SQUID. A pulse from an X-ray that hits one TES is visible in every SQUID channel.

The SQUIDs can be demultiplexed via the inverse matrix to recover the original TES signal.

Outlook

Error correction codes provide robustness against single SQUID failures for next-generation TES arrays with high multiplexing factors. The ability of a system to survive one or more simulated SQUID failures with minimal loss of energy resolution has been demonstrated using data from a 32 channel TES array.

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Error Correcting Codes

High multiplexing factors increase the risk of failure: a single inoperable SQUID leads to an under-constrained system of equations. Error correction is needed to ensure that a system can survive one or more SQUID failures. The basic CDM de-multiplexing equation can be written:

$$T_\nu = D_{\nu\lambda} S_\lambda$$

Where *T* is a vector of TES signals, *D* is the decoding matrix, and *S* is a vector of SQUID signals. Summation is implicit over repeated indices. If some SQUID, S_a is inoperable, the TES signals can no longer be reconstructed. If one TES, T_b has a known value, we can write

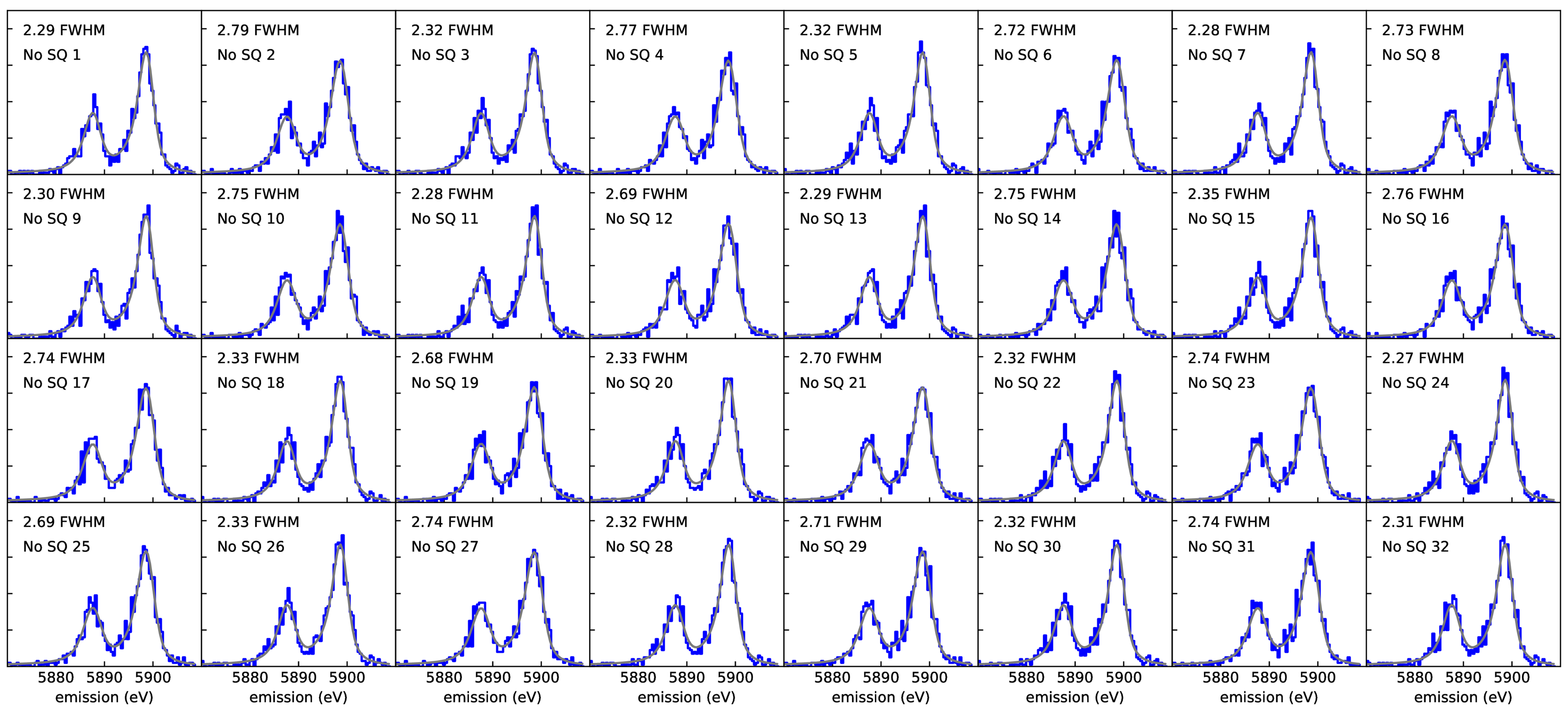
$$T_b = D_{b\lambda} S_\lambda = \sum_{\lambda \neq a} D_{b\lambda} S_\lambda + D_{ba} S_a$$

And solve for S_a using the rest of the *S* vector.

$$S_a = \frac{T_b - \sum_{\lambda \neq a} D_{b\lambda} S_\lambda}{D_{ba}}$$

Then the entire TES vector can be recovered, since the SQUID vector is now complete. Any single SQUID loss is recoverable as long one TES is known. In practice, the TES T_b could be "virtual", i.e, if 32 SQUIDs are used to read out 31 TES detectors, there is one "virtual" TES with zero signal.

Error Correcting Code Demonstration



We have tested this algorithm using data from a 32 channel CDM system. 16 active TES sensors were bombarded with Mn K-α emission and read out using 32 SQUIDs. The remaining 16 TES channels were disconnected. We can treat one of the 16 disconnected TES as a "fixed" signal T_b and then eliminate any one of the 32 SQUIDs. We use our ECC algorithm to correct for the dropped SQUID and obtain a high-quality Mn K-α spectrum. The data above show the spectrum from one detector as each possible SQUID is "disabled" in software and then reconstructed using our ECC algorithm.

Correcting Multiple SQUID failures

In principle, *n* SQUID failures can be corrected in the same fashion by using *n* known TES signals to solve a linear system of equations to reconstruct the SQUID signals.

$$T_{b_n} - \sum_{\lambda \neq a_1 \ldots a_n} D_{b_n \lambda} S_\lambda = \sum_{a_n} D_{b_n a_n} S_{a_n}$$

$$\sum_{b_n} D^{-1}_{a_n b_n} \left(T_{b_n} - \sum_{\lambda \neq a_1 \ldots a_n} D_{b_n \lambda} S_\lambda\right) = S_{a_n}$$

In practice, we see that there is a sub-matrix of the decoding matrix *D* that has to be inverted to reconstruct the SQUIDs. If this sub-matrix is invertible, the SQUID signals can be recovered. We demonstrate multi-SQUID correction in the plots below, which come from the same data set as the single-failure plots