Microwave Kinetic Inductance Detector for GroundBIRD

Microwave Kinetic Inductance Detector (MKID) is a photon detector using resonators in superconducting state. When microwave photons are absorbed by MKID, their energy break Cooper pairs and change the inductance of the resonator. The intensity of microwave can be measured with changes in amplitude and phase of the frequency comb which is a combination of I/Q signals at the resonant frequencies of the MKIDs. We will use this type of photon detectors for GroundBIRD, a CMB B-mode polarization experiment.

Readout system

We have developed a readout system for our MKID array. An FPGA (Field Programmable Gate Array) evaluation board and the RHEA analog board are used. It generates a frequency comb and the frequency of each channel can be changed individually whenever we need. The shifted frequency comb from MKID array is separated to each channel in channelization process, and down-sampled to remove cross-talks.

Leakage problem by cross-talk signals

Our readout system can use the arbitrary frequencies and it gives many benefits. However, it has a potential problem: leakage. After channelization, we can get I/Q signals of N channel as follows.

\[ I'_j = I_j + Q_j\cos\phi_j - Q_j\sin\phi_j, \]
\[ Q'_j = I_j + Q_j\sin\phi_j + Q_j\cos\phi_j, \]
where \( A_j \) is the amplitude of the \( j \)th channel of the signal to DAC, \( A_j \) and \( \phi_j \) are the amplitude and phase shift of \( j \)th channel of DAC output. These signals are consisted with two terms: \( j \)th channel term and cross-talk term that came from other channels.

Case1: The frequencies of cross-talk terms are integer multiples of the sampling frequency \( f_{\text{Sample}} \).

\[ \omega_j - \omega_n = 2\pi f_{\text{Sample}} n \]
In this case, cross-talk terms can be completely removed by time average over a proper length of time window (Fig. 3).

\[ \bar{I}_j = \frac{1}{T} \int_{t}^{t+T} I_j(t) dt = A_j \cos \phi_j \]
\[ \bar{Q}_j = \frac{1}{T} \int_{t}^{t+T} Q_j(t) dt = A_j \sin \phi_j \]

Case2: If the frequency intervals don’t satisfy the condition of case 1, down-sampling leaves leakages, \( h_{dj} \) and \( \eta_{dj} \) (Fig. 4).

\[ \bar{I}_j = \frac{1}{T} \int_{t}^{t+T} I_j(t) dt = A_j \cos \phi_j + h_{dj} \left( \sin\omega_j t - \sin\omega_n t \right) \]
\[ \bar{Q}_j = \frac{1}{T} \int_{t}^{t+T} Q_j(t) dt = A_j \sin \phi_j + h_{dj} \left( \cos\omega_j t - \cos\omega_n t \right) \]
These terms make errors in amplitude and phase shift (Fig. 5).

Leakage error depends on the frequency interval \( \Delta f \) between channels, number of channels and sampling rate (DAQ speed). In our system, the error is 1.5% for \( \Delta f = 0.1 \) MHz and 0.25% for \( \Delta f = 1 \) MHz with the sampling rate of 10 kHz and 2 channel configuration.

Conclusions

- Our readout for GroundBIRD can use arbitrary frequencies regardless of the sampling frequency.
- Generally, cross-talk signals leave leakage in down-sampling process, result error in amplitude and phase shift.
- This problem can be solved by applying window function.
- Our logical test shows that Hannig window removes almost of the leakage error and improves measurement accuracy.

Window function

To eliminate the leakage, we multiply an window function \( w(n) \), Hanning window, to the ADC signals, \( I' \) and \( Q' \), before the channelization (Fig. 6).

\[ I_{\text{in}} = I' \times w(n), \quad Q_{\text{in}} = Q' \times w(n) \]
Hanning window, \( w(n) = \sin^2 \left( \frac{\pi n}{L} \right) \), \( L \) is the length of window.

The window function is implemented in the FPGA with the block memory. The length of the window function is 10,000 samples. We can change the window length by up/down sampling. The error from this scaling is small enough compared with the bit error of the ADC data, so we can use this method without interpolations.

The other way to reduce the leakage is using large down-sampling rate or wider interval between resonances. However these methods degrades the sampling rate or number of resonators in a given frequency range. Our method is the unique solution to increase sampling rate or frequency of leakage.

Since the signal loses half of information by multiplying window function, noise from other sources can be increased by applying it.

Test and Result

We tested our system with Hannig window in algorithmic level by using Matlab and real FPGA loop. For FPGA loop test, we connect the DAC signal to the ADC signal inside the FPGA. It was tested with no analog parts.

The sampling frequency for this test is 10 kHz. It is determined by the ADC frequency of 200 MHz and down-sampling rate of 20,000 samples.

The \( \Delta f \) is frequency intervals between neighboring channels in frequency domain. It is set to a regular value, 1 MHz (Case1), to check the ideal case without leakage. For \( \Delta f = 1.003 \) MHz which is not an integer multiple of 10 kHz (Case2), we will have leakage error without tuning of sampling rate. We also tested irregular intervals to consider more realistic case (Case3). The errors in amplitude and phase shift with and without Leakage windows were compared.

When the leakage occurs, the window function eliminates the leakage and improves the accuracy of measurement.

Matlab

<table>
<thead>
<tr>
<th>Channel Number</th>
<th>No window</th>
<th>Hannig window</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amplitude error ( \sigma_A \times 10^4 )</td>
<td>65</td>
<td>45</td>
</tr>
<tr>
<td>Phase error ( \sigma_{\phi} \times 10^5 )</td>
<td>20</td>
<td>30</td>
</tr>
</tbody>
</table>

Real FPGA loop

<table>
<thead>
<tr>
<th>Channel Number</th>
<th>No window</th>
<th>Hannig window</th>
</tr>
</thead>
<tbody>
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