Development of a cross-talk suppression algorithm for MKID readout

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Microwave Kinetic Inductance Detector for GroundBIRD

Microwave Kinetic Inductance Detector (MKID) is a photon detector using resonators in superconducting state. When microwave photons are absorbed by MKID, their energy break Cooper pairs and change the inductance of the resonator. The intensity of microwave can be measured with changes in amplitude and phase of the frequency comb which is a combination of I/Q signals at the resonant frequencies of the MKIDs. We will use this type of photon detectors for GroundBIRD, a CMB B-mode polarization experiment.

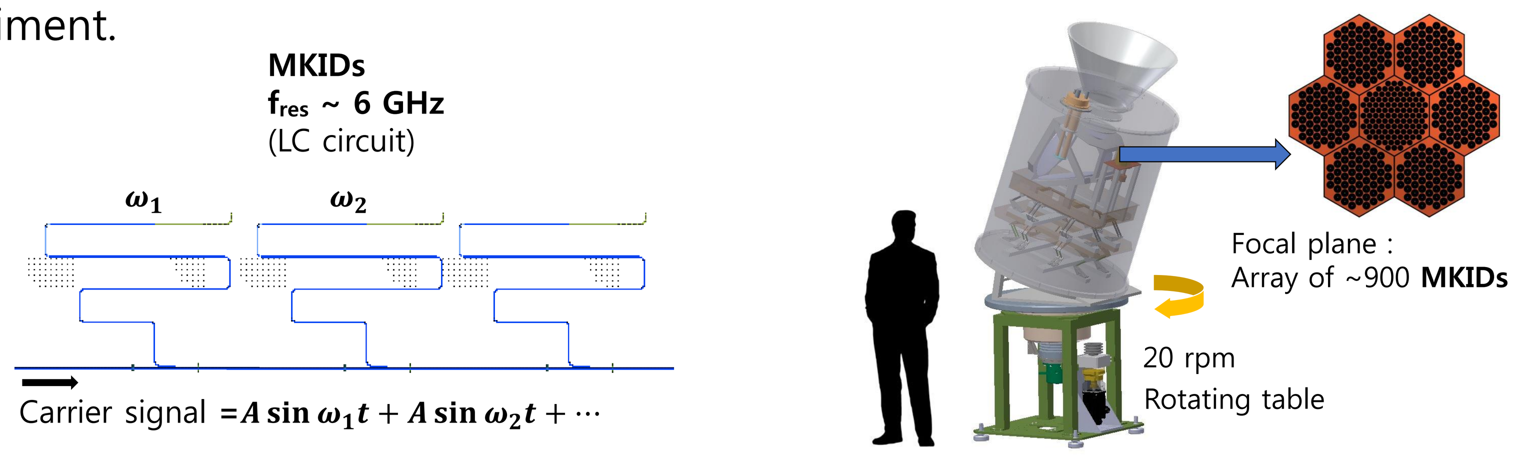

Figure 1. An MKID array and the GroundBIRD experiment. GroundBIRD uses MKID as photon detector.

Readout system

We have developed a readout system for our MKID array. An FPGA (Field Programmable Gate Array) evaluation board and the RHEA analog board are used. It generates a frequency comb. and the frequency of each channel can be changed individually whenever we need. The shifted frequency comb from MKID array is separated to each channel in channelization process, and down-sampled to remove cross-talks.

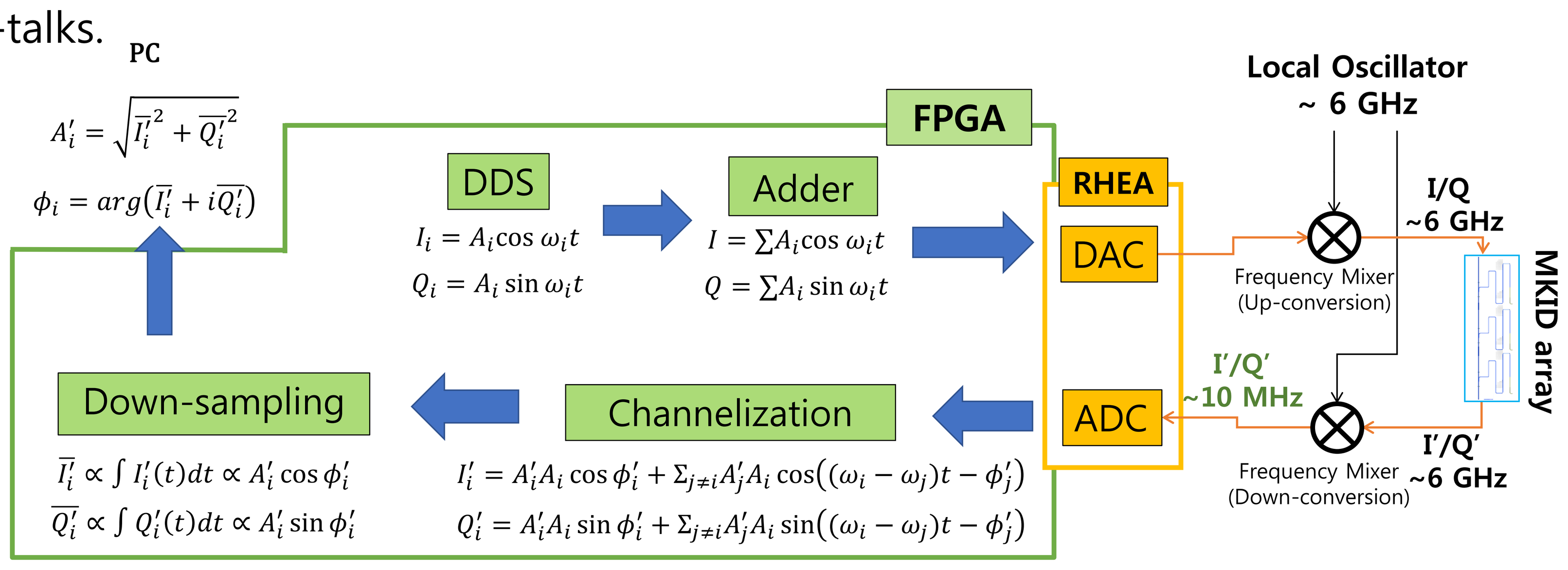

Figure 2. Schematic of the Readout system.

Leakage problem by cross-talk signals

Our readout system can use the arbitrary frequencies and it gives many benefits. However it has a potential problem: leakage.

After channelization, we can get I/Q signals of N channel as follows.

$$I_i' = I'I_i + Q'Q_i = A_i'A_i \cos\phi_i' + \Sigma_{j\neq i} A_j'A_i \cos\big((\omega_i - \omega_j)t - \phi_j'\big)$$

$$Q_i' = Q'I_i - I'Q_i = A_i'A_i \sin\phi_i' + \Sigma_{j\neq i} A_j'A_i \sin\big((\omega_i - \omega_j)t - \phi_j'\big)$$

where A_i is the amplitude of the i^{th} channel of the signal to DAC, A_i' and ϕ_i' are the amplitude and phase shift of i^{th} channel of ADC output. These signals are consisted with two terms: i^{th} channel term and cross-talk terms that came from other channels.

Case1: The frequencies of cross-talk terms are integer multiples of the sampling frequency $f_{sampling}$,

$$\omega_i - \omega_j = 2\pi n\, f_{sampling}\,, \qquad n \text{ is an integer.}$$

In this case, cross-talk terms can be completely removed by time average over a proper length of time window (Fig. 3).

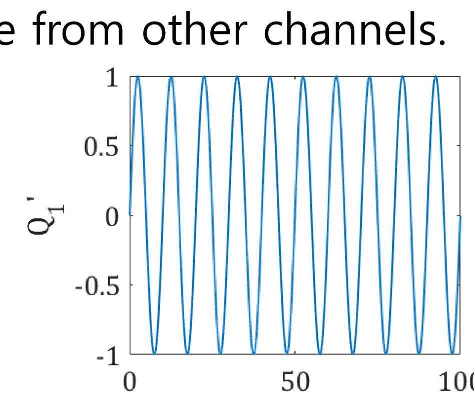

Figure 3. Signal without leakage for frequency difference Δf = 0.1 MHz

$$\overline{I_i'} = \frac{1}{A_i}\frac{1}{t_1 - t_0}\int_{t_0}^{t_1} I_i'(t)dt = A_i' \cos\phi_i'$$

$$\overline{Q_i'} = \frac{1}{A_i}\frac{1}{t_1 - t_0}\int_{t_0}^{t_1} Q_i'(t)dt = A_i' \sin\phi_i'$$

Case2: If the frequency intervals don't satisfies the condition of case1, down-sampling leaves leakages, $h_{I,i}$ and $h_{Q,i}$ (Fig. 4).

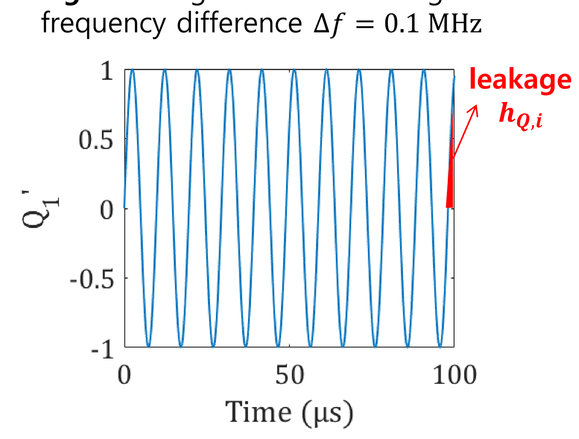

Figure 4. Signal with leakage for Δf = 0.102 MHz. This leakage makes 1% error in amplitude and phase shift.

$$\overline{I_i'} = \frac{1}{A_i}\frac{1}{t_1 - t_0}\int_{t_i}^{t_f} I_i'(t)\,dt = A_i' \cos\phi_i' + h_{I,i} \left(= \Sigma_{j\neq i}\frac{1}{t_1 - t_0}\int_{t_0}^{t_1} A_j' \cos\left[(\omega_i - \omega_j)t - \phi_j'\right] dt\right)$$

$$\overline{Q_i'} = \frac{1}{A_i}\frac{1}{t_1 - t_0}\int_{t_0}^{t_1} Q_i'(t)\,dt = A_i' \sin\phi_i' + h_{Q,i} \left(= \Sigma_{j\neq i}\frac{1}{t_1 - t_0}\int_{t_0}^{t_1} A_j' \sin\left[(\omega_i - \omega_j)t - \phi_j'\right] dt\right)$$

These terms make errors in amplitude and phase shift (Fig. 5). Leakage error depends on the frequency interval Δf between channels, number of channels and sampling rate (DAQ speed). In our system, the error is 1.5% for Δf=0.1 MHz and 0.25% for Δf =1 MHz with the sampling rate of 10 kS/s and 2 channel configuration.

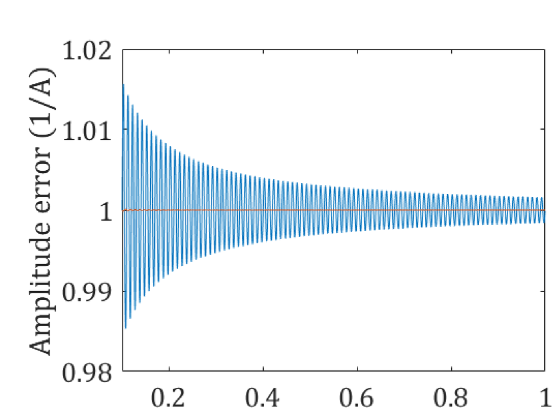

Figure 5. Relative Amplitude error for Δf = 0.1 ~ 1 MHz.

Case3: In realistic case, the resonant frequencies of MKIDs are unequally spaced, and the frequency intervals are arbitrary numbers. In this case, leakage error will be random and unpredictable.

Window function

To eliminate the leakage, we multiply an window function $w(n)$, **Hanning window**, to the ADC signals, I' and Q', before the channelization (Fig. 6).

$$I'_{win} = I' \times w(n), \qquad Q'_{win} = Q' \times w(n)$$

$$\text{Hanning window}, w(n) = \sin^2\left(\frac{\pi n}{L-1}\right), \qquad L \text{ is the length of window.}$$

The window function is implemented in the FPGA with the block memory. The length of the window function is 10,000 samples. We can change the window length by up/down sampling. The error from this scaling is small enough compared with the bit error of the ADC data, so we can use this method without interpolations.

The other way to reduce the leakage is using large down-sampling rate or wider interval between resonances. However these methods degrades the sampling rate or number of resonators in a given frequency range. Our method is the unique solution to increase sampling rate or degree of multiplexing.

Since the signal loses half of information by multiplying window function, noise from other sources can be increased by applying it.

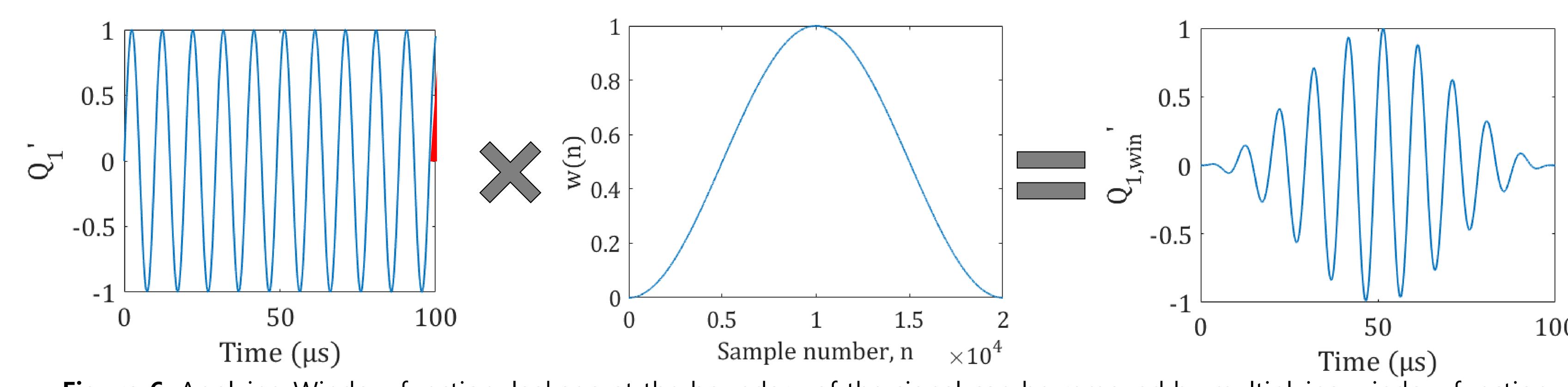

Figure 6. Applying Window function. leakage at the boundary of the signal can be removed by multiplying window function.

Test and Result

We tested our system with Hanning window in algorithmic level by using Matlab and real FPGA loop. For FPGA loop test, we connect the DAC signal to the ADC signal inside the FPGA. It was tested with no analog parts.

The sampling frequency for this test is 10 kS/s. It is determined by the ADC frequency of 200 MHz and down-sampling rate of 20,000 samples.

The Δf's are frequency intervals between neighboring channels in frequency domain. It is set to a regular value, 1 MHz (Case1), to check the ideal case without leakage. For Δf=1.003 MHz which is not an integer multiple of 10 kS/s (Case2), there will be leakage error without tuning of sampling rate. We also tested irregular frequency intervals to consider more realistic case (Case3). The errors in amplitude and phase shift with and without window function were compared.

When the leakage occurs, the window function eliminates the leakage and improves the accuracy of measurement.

Matlab	Amplitude error, σ_A/A ($\times 10^5$)		Phase error, σ_ϕ(rad, $\times 10^5$)	
	No window	Hanning window	No window	Hanning window
Case1: Regular Δf=1 MHz (no leakage expected)	24	26	45	91
Case2: Regular Δf=1.003 MHz	21	4.0	2700	3.4
Case3: Irregular Δf	1650	10.3	2500	10.2

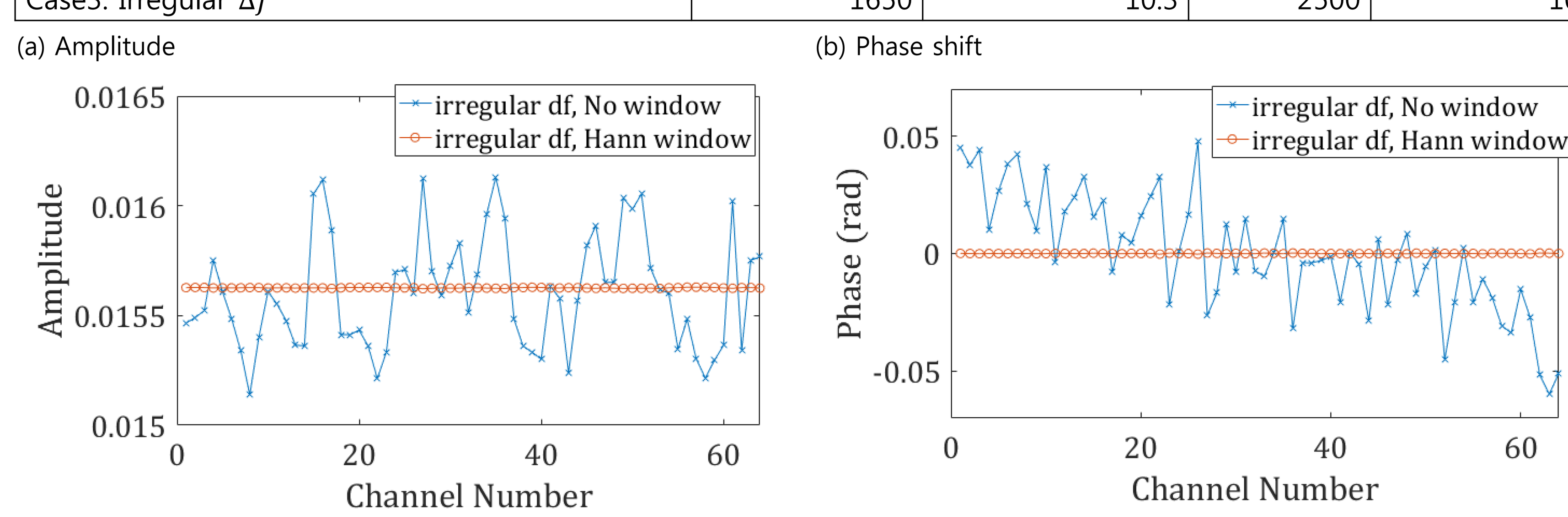

Figure 7. Amplitude and phase shift w.r.t. the channel number for Case3 (Matlab test). Hanning window suppresses the leakage by 1/20 for in amplitude and phase shift.

Real FPGA loop	Amplitude error, σ_A/A ($\times 10^5$)		Phase error, σ_ϕ(rad, $\times 10^5$)	
	No window	Hanning window	No window	Hanning window
Case1: Regular Δf=1 MHz (no leakage expected)	102	22	269	20
Case2: Regular Δf=1.003 MHz	55	3.2	271	3.5
Case3: Irregular Δf	1657	9.9	2512	9.8

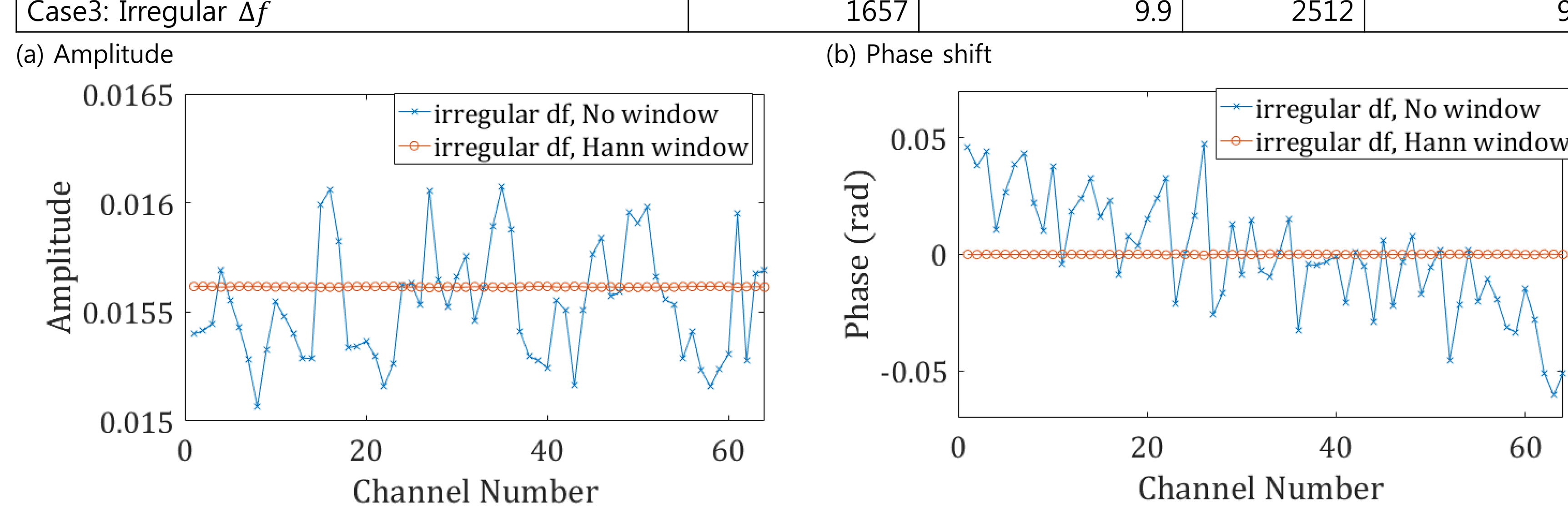

Figure 8. FPGA loop test for irregular frequency interval (Case3). Hanning window improves the accuracy of amplitude and phase shift. It was tested logically without any analog parts.

Conclusions

- Our readout for GroundBIRD can use arbitrary frequencies regardless of the sampling frequency.
- Generally, cross-talk signals leave leakage in down-sampling process, result error in amplitude and phase shift.
- This problem can be solved by applying window function.
- Our logical test shows that Hanning window removes almost of the leakage error and improves measurement accuracy.

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